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# **Choice of The Fiscal Rules in The Long Run**

# Abstract

Paper examines the problem of choice of an optimal fiscal rule in the long run. An ideal rule would typically assure fair distribution of utility over generations, while allowing to maintain the sustainable fiscal position. Three commonly used types: debt, deficit and expenditure rules are considered. The main conclusion is that only the modified deficit rule fulfils the assumptions. The rule requires that government's policy should aim at keeping the debt-to-GDP ratio constant over the economic cycle.

# Introduction

One of the key issues in an intensive discussion over the fiscal policy in the last few years is the persistence of public finance deficit that has been observed in the majority of industrial countries. Many economists treat such a situation as a signal of an excessive looseness of public finances, posing threat to the medium- and long run financial sustainability. Data for years 1992–2003 indicates that a similar problem emerged in Poland – public finance deficit could be observed over the whole period. The phenomenon persisted even in the times of economic revival in the years 1995–1998, when the rapid growth of public income created an opportunity to achieve surplus and public debt reduction, which would leave a security margin for the following years. Economic downturn that has been observed since 2000 clearly revealed the problem of excessive expenditure as compared to the state financial capabilities. The excessive expenditures level imposed the necessity to reduce them drastically under the recession conditions, that is when economic theory suggests rather an expansionary policy. The above mentioned problems indicate an urgent need to subordinate the fiscal policy to the defined medium- and long run objectives. One of such ways is introducing appropriate institutional solutions which will help to confine the expenditure growth and increase the awareness the intertemporal public sector budget constraint among the policymakers. The significance of institutional factors influencing the fiscal policy is supported by the research of such authors as von Hagen (1992), Alesina and Perotti (1994), Milesi-Ferretti (1997), mainly based on international comparisons. Alesina, Perotti (1994) and Milesi-Ferretti (1997) define the fiscal institutions as all the rules and regulations according to which budgets are drafted, approved and implemented. The above mentioned statistical research indicate that such factors as strong prerogatives given to the Minister of Finance, a limited range of parliamentary amendments to the budget act or the procedures limiting the flexibility of budget execution may contribute to decrease of deficit.

In the cited papers fiscal policy rules are mentioned among the most important institutions, that, if properly constructed, may have strong influence on fiscal policy. Milesi-Ferretti defines the fiscal policy rules as all the constraints imposed on deficit, public debt or spending levels that influence budget drafting or execution. Kopits and Symansky (1998) additionally assume that a crucial condition is the permanent character of the constraint. Though this last assumption is not necessary, in this study the permanent character of the fiscal rules is assumed.

Discussion over applying rules in economic policy has been present in the economic literature for a long time. Among the earliest arguments for "rules rather than discretion" is the research of Kydland and Prescott (1997) concerning the time inconsistency problem. One of the arguments for applying rules is the problem of "long and variable lags" between an event (e.g recession) and the policy reaction if the discretionary framework is applied (Stiglitz 1988). Another argument has been raised in the context of fiscal policy under the currency union - common currency allows for shifting consequences of irresponsible fiscal policy to other countries. In this situation the existence of rules may provide the necessary coordination of policy conducted by the member countries. Under the Economic and Monetary Union such a role is fulfilled by the Maastricht fiscal criteria and the Stability and Growth Pact. The analysis of problems connected with conducting fiscal policy under economic the union can be found in study by Brunila, Buti and Franco (2001).

The last argument in favour of rules in fiscal policy is connected with the new political economy. Theoretical and empirical research show that in democratic countries fiscal policy suffer from bias towards excessive expenditure and deficit levels, which in unfavourable conditions may even lead to the country's insolvency. An overview of theoretical models explaining roots of this phenomenon can be found in work by Alesina and Perotti (1994) Imposing adequate rules may limit the scope of this phenomenon, forcing the politicians to conduct fiscal policy that is close to the social optimum in the long run.

On the other hand, arguments for discretionary policy stress that it is hard to define a rule which would both comply with the long-term economic policy objectives and allow for reaction to shocks. According to some opinions, though rules have their advantages, excessive restrictions may hamper an effective anticyclical economic policy. This argument is especially important as related to EMU, where introducing the common currency caused the fiscal instruments to remain the only means of demand stabilization.

A significant part of discussion over fiscal rules concerns the problem of choice of an optimum fiscal rule. Requirements concerning an optimum fiscal rule have been analysed by Kopits, Symansky (1998) and Buiter (2003). They can be divided into three main groups.

- An optimum fiscal rule should efficiently influence the fiscal policy. It means that among others it should be simple, and the accordance of policy with rule should be evident and easy to control.
- The accepted rule should comply with the defined long run objectives.
- It should also enable efficient anti-cyclical policy allow for increasing budget deficit during a recession and force its reduction (or an occurrence of surplus) during upswings.

The analysis presented in this paper concentrates on long-term aspects of optimum fiscal rule choice. This article aims to study which fiscal rules may be applied in a long run and which of them to the greatest extent contributes to increasing social welfare. The basic model is developed by including the existence of public capital, which allows to account for the distinctive features of developing countries, such as higher public investment needs.

Section one of the study defines the structure and assumptions of the examined model of public income and expenditure. The objective of section two is to answer what expenditure and deficit level provide solution that is optimal in the long run. In section three possible reasons for which the real fiscal policy may in reality deviate from optimum are briefly examined. Section four is an attempt to compare the results of applying particular fiscal rules in the long run perspective. Section five concludes the analysis.

# **1.** Assumptions of the model

A number of simplifying assumptions was introduced in the following analysis. The aim of these simplifications is to emphasize those aspects of fiscal policy which refer to the long run aspects.

- State income and expenditure are income and expenditure of the consolidated public sector, without distinction between central budget, extrabudgetary funds and local governments.
- Public debt includes all public liabilities, including risk-weighted guarantees. The debt is denominated in local currency and bears interest according to the fixed long-run interest rate. The distinction between net and gross debt is omitted here these two categories are treated as identical.
- Full transparency of fiscal policy is assumed, which means inability to apply "creative accounting" techniques. Consequent on that, formal accordance of fiscal policy with a given rule equals its actual accordance. A broader analysis of influence of transparency on applying the fiscal rules is presented by Milesi-Ferretti (2000).
- The problem of who imposes the fiscal rule is omitted here. It may be e.g. the parliament which obliges the government to present the budget draft which is in accordance with the rule. Fiscal rules may also take the form of self-constraints imposed by the government their objective may in this case be decreasing risk connected with possible state's insolvency.
- This study emphasizes examining long-term consequences of various types of fiscal policy, thus effects connected with the business cycle are omitted. In the consequence of such approach, all presented variables should be interpreted as net of cyclical effects. It is assumed that within the full business cycle the sum of cyclical effects is zero.

It is assumed that primary government expenditure (i.e. expenditure net of interest on public debt) are the source of social utility associated with the existence of public sector. Primary expenditure (in real terms) G consist of transfers to the private sector  $G_T$ , government consumption  $G_C$  and capital expenditure (investment)  $G_I$ , so that:  $G = G_T + G_C + G_L^{-1}$ . Nominal values obtained by multiplying real values by prices level P; in the long run prices

<sup>&</sup>lt;sup>1</sup> All variables are assumed to be of the class at least  $c^{(2)}$  at least. A convention is assumed in which the variable X(t) is substituted by X. Where it is not *explicite* stated that a variable is a fixed parameter, it should be assumed that it may be a certain function of time. Notation  $\dot{\mathbf{x}}$  denotes the growth of variable, i.e.  $\dot{\mathbf{x}} = \partial \mathbf{x} / \partial t$ , where t denotes time. The relative level of variable X denotes its ratio to the gross domestic product.

increase at the constant rate<sup>2</sup>  $\dot{P}/P = \pi$ . Total public expenditure consists of primary expenditure and public debt service costs equal to the product of debt B and the nominal interest rate (r +  $\pi$ ), where r is the real interest rate.

The source of expenditure financing is tax income T, being the product of gross domestic product Y and a flat tax rate  $\tau$ . In the long term income rises at a constant real rate n, the same as the long-term real growth rate of GDP. Thus, the nominal GDP and nominal income (PT) rise in the long run with the rate (n +  $\pi$ ). The difference between total real expenditure and income equals the deficit D: D = G + rB - T, financed by loans drawn on the capital market. Nominal deficit PD equals the increase of nominal public debt PB: PD =  $\partial$ PB/ $\partial$ t. It can be shown that real debt growth equals the difference between the real deficit and debt reduction caused by inflation:  $\dot{B} = D - \pi B$ .

The equation of movement of the public debt, being a dynamic form of intertemporal budget constraint is then:

$$(1) \qquad B = G + rB - T.$$

Crucial for subsequent analysis is the assumption that the considered state is solvent in the long run. Buiter (1998) shows that necessary and sufficient condition for solvency is that the average long-run public debt growth rate is lower that the average long-run interest rate. In the other words, it is assumed that the country does not finance its expenditure by loans drawn to cover the costs of servicing the previous ones, or does not use Ponzi financing. Formally this condition can be written as

(2) 
$$\lim_{t\to\infty} B(t)e^{-rt} = 0$$

Transforming the equation of movement of the public debt we arrive at G-T = B - rB. By multiplying both sides by factor  $e^{-rt}$  and integrating we arrive at:

(3) 
$$\int_{0}^{\infty} (G-T)e^{-rt}dt = \int_{0}^{\infty} \dot{B}e^{-rt}dt - \int_{0}^{\infty} rBe^{-rt}dt.$$

Integrating the term Be<sup>-rt</sup> it may be proved that

(4) 
$$\int_{0}^{\infty} \dot{B}e^{-rt}dt = \int_{0}^{\infty} rBe^{-rt}dt + [Be^{-rt}]_{0}^{\infty}.$$

Substituting 1 to 1 and subsequently to 1, we arrive at the long-term form of intertemporal budget constraint:

 $<sup>^2</sup>$  In the further analysis symbol  $\dot{x}$  denotes marginal increase, so  $\dot{x}=\partial x\,/\,\partial t\,$  holds, where t stands for time.

(5) 
$$\int_{0}^{\infty} Ge^{-rt} dt + B(0) = \int_{0}^{\infty} Te^{-rt} dt,$$

where b(0) is the initial level of public debt (in period t = 0). Thus, the long-run budgetary constraint is satisfied if the sum of future discounted income T is sufficient to finance the sum of future discounted primary expenditure G and the initial public debt level B(0).

# 2. Choice of the optimum fiscal policy

In this section an attempt is made to define the optimal fiscal policy, which can be the point of reference in the later part of analysis, for the assessment of the specific fiscal rules. Two alternative criteria are considered: one based on standard utility function and the other, simplified, based on the assumption concerning the constant share of expenditure in GDP. The policy which fulfils the optimality criterion and satisfied also the intertemporal budget constraint, will be referred to as socially optimal fiscal policy.

#### 2.1. The utility function-based approach

This approach assumes that the objective of fiscal policy in the analysed model is maximizing long-run social utility of government expenditure U<sub>G</sub>, equal the sum of future discounted momentary utilities  $u_G(t)$ :  $U_G = \int_0^{\infty} u_G(t)e^{-\rho t}dt$ , where  $\rho > 0$  is the discount rate of future utility. It is assumed that momentary utility  $u_G(t)$  is the growing function of the primary government expenditure, with decreasing marginal utility. In the presented model we assume the widely used constant relative risk aversion (CRRA) utility function<sup>3</sup>  $u_G(t) = \frac{G(t)^{1-\nu} - 1}{1-\nu}$ . Thus, the objective of the government is to maximize the following long run utility function:

(6) 
$$U_G = \int_0^\infty \frac{G(t)^{1-\nu} - 1}{1-\nu} e^{-\rho t} dt$$

<sup>&</sup>lt;sup>3</sup> It should be noticed that at values of parameter  $\nu$  close to 1 the utility function CRRA converges to the logarithmic utility function.

The optimalisation problem is to find the expenditure path G(t) that maximizes, while satisfying the budgetary constraint 1 (or its dynamic form 1). To obtain the solution the Pontriagin's extremum rule is applied (after: Tokarski 2001) This method is to a great extent analogous with a more common Lagrange's procedure of finding the conditional extremum. Its basic instrument is the current-value Hamiltonian  $H = u_G(t) + \theta B$ , which in this case equals:

(7) 
$$H = \frac{G^{1-\nu} - 1}{1-\nu} + \theta(rB + G - T),$$

where  $\theta$  is a price being an equivalent of Lagrange's  $\lambda$  multiplier. The optimal path G(t) has to fulfil the following necessary conditions:

- (8)  $\partial H/\partial G = G^{-\nu} + \theta = 0$ ,
- (9)  $-\partial H/\partial B + \rho \theta = -\theta r + \rho \theta = \dot{\theta}$ ,
- (10)  $\partial H / \partial \theta = rB + G T = \dot{B}$ ,

and the transversality condition

(11)  $\lim \theta e^{-\rho t} = 0$ 

It follows from that  $G^{-\nu} = -\theta$ . It can easily be shown that the multiplier  $\theta$  must be negative and  $\nu(G/G) = -\theta/\theta$  holds. From the equation it follows that the multiplier  $\theta$  grows with the fixed rate  $\dot{\theta}/\theta = \rho - (r + \pi)$ , thus:

(12) 
$$G/G = (r-\rho)/\nu$$
.

It has been shown that the expenditure G grows in the long run with the fixed rate equal the difference between real interest rate and discount rate, multiplied by the parameter v. Thus, the differential equation defines a group of expenditure paths. If we assume that at the period 0 the primary expenditure amounts to G(0), then solving the equation yields  $G(t) = G(0)e^{((r-p)/v)t}$ . It may be shown that the transversality condition is fulfilled for each path, since:

(13) 
$$\lim_{t\to\infty} \theta e^{-\rho t} = \lim_{t\to\infty} (-1/G(0)) e^{(\rho-r)t} e^{-\rho t} = \lim_{t\to\infty} (-1/G(0)) e^{-rt} = 0,$$

which is always true if the long-run real interest rate is positive.

The next step is to choose such an expenditure growth path G(t) which satisfies the intertemporal budget constraint 1. Basing on the assumptions we know that income T increases with the fixed rate equal the GDP growth rate:  $T(t) = T(0)e^{nt}$ . From the previous considerations we also know that the expenditure G increases with the fixed rate  $(r-\rho)/\nu$ . It should be noticed that if

the government expenditure and product growth rate differed in the very long term (with  $t\rightarrow\infty$ ), the share of government expenditure in the product would increase to 100% or decrease to 0. In actual economies this tendency cannot be observed in the long run, hence, it can be assumed that, at least in the long run, the growth rates of primary expenditure and product are equal. It means that  $\lim_{t\rightarrow\infty} \dot{G}/G = n$  must hold. However, since the expenditure growth rate G that maximizes the long-run social utility is constant, the equation  $(r-\rho)/\nu = n$  holds in each period. Therefore, the budget constraint can be defined as:

(14) 
$$\int_{0}^{\infty} G(0)e^{(n-r)t}dt + B(0) = \int_{0}^{\infty} T(0)e^{(n-r)t}dt$$

or, after transformations as

(15) 
$$\frac{G(0)}{r-n} + B(0) = \frac{T(0)}{r-n}$$

Let us define the relative public primary expenditure level g as its ratio to the gross domestic product: g = G/Y. It follows from the previous considerations that the expenditure increases in the long run with the same rate as GDP, hence this relation is constant, i.e.  $g(t) \equiv g(0)$ . Similarly, the relative tax level  $\tau$  is equal to the ratio of T to GDP. In relative categories the above equation may be defined as:

(16) 
$$\frac{g}{r-n} + b(0) = \frac{\tau}{r-n}$$
.

Thus, the relative primary expenditure level that maximizes the social utility, which at the same time satisfies the budget constraint (the socially optimal level) equals:

(17) 
$$g_{s}^{*} = \tau - (r - n)b(0)$$

Let us analogically define the relative debt and deficit level b and d, respectively, as its share of GDP. As it was shown earlier,  $d = g + (r + \pi)b - \tau$  holds, therefore the socially optimal deficit level is given by:

(18) 
$$d_{\rm S}^* = (n + \pi)b_{\rm L}$$

Thus, it equals the product of the relative public debt level and the nominal rate of economic growth. It may be proved that the socially optimal fiscal policy means setting the expenditure and deficit in such a way that the ratio of public debt to GDP remains constant. It follows from the previous considerations that  $B = D - \pi B$ . Using the derivative formula, we may calculate the growth of the relative level of public debt b as:

(19) 
$$\dot{\mathbf{b}} = \partial \left(\frac{\mathbf{B}}{\mathbf{Y}}\right) / \partial \mathbf{t} = \frac{\dot{\mathbf{B}}\mathbf{Y} - \mathbf{B}\dot{\mathbf{Y}}}{\mathbf{Y}^2} = \frac{\dot{\mathbf{B}}}{\mathbf{Y}} - \frac{\dot{\mathbf{Y}}}{\mathbf{Y}}\frac{\mathbf{B}}{\mathbf{Y}} = \frac{\mathbf{D}}{\mathbf{Y}} - \pi\frac{\mathbf{B}}{\mathbf{Y}} - \frac{\dot{\mathbf{Y}}}{\mathbf{Y}}\frac{\mathbf{B}}{\mathbf{Y}}$$

Thus, it holds:

(20)  $b = d - (n + \pi)b$ .

The above formula means that the growth of the relative level of public debt equals the relative deficit, net of product of debt level and nominal economic growth rate. Comparison of the formulas and demonstrates that if the government conducts the socially optimal fiscal policy that maximizes the social utility, then the public debt remains constant (i.e.  $\dot{b} = 0$ ) in relation to GDP.

As an example let us assume a relative initial public debt level of 40% and an average tax rate  $\tau = 40\%$ . The long-run real economic growth rate is 2%, the nominal is 4% and the real interest rate equals 4%. In such case the socially optimal (fixed) deficit level will be  $d_s^* = (n + \pi)b = 4\% * 40\% = 1,6\%$ . The corresponding long-run share of primary expenditure in product will be  $g = \tau + d_s^* - (r + \pi)b = 39,2\%$ , and the share of overall expenditure will be  $g = \tau + d_s^* - (r + \pi)b = 39,2\%$ . At these levels of expenditure and deficit the public debt will remain at the constant relative level 40%.

#### 2.2. The simplified approach

It appears that the approach based on the utility function is characterized by considerable limitations. First, the chosen utility function considerably affects the result. In most cases it is arbitrary and does not have to reflect the real social preferences. Moreover, as it was proved, to find a reasonable long-run solution we need to make a strong assumption with values of parameters v so that the long-run expenditure growth rate equals GDP growth rate.

For these reasons an alternative approach to the optimality problem might be suggested. It is based on the a priori assumption that socially optimal fiscal policy should be characterized by a constant share of utility (primary) public expenditure in the gross domestic product (g=const.). Let us now analyse what consequences the above assumption implies for the long run variables paths such as the deficit or public debt.

According to the definition of deficit and equation , the following identity holds:

(21)  $b = (r-n)b + (\tau - g)$ .

With then assumption that the long run interest rate and the economic growth rate are different and constant, it is a differential equation. According to Chiang (1984,) the solutions of this equation is expressed by:

(22) 
$$b(t) = \frac{g-\tau}{r-n} + \left(w(0) - \frac{g-\tau}{r-n}\right)e^{(r-n)t}$$

As presented earlier, maintaining solvency in the long run does not require fulfilling the condition 1. Comparing it with the above formula we may prove that the country is solvent in the long run only when

(23) 
$$\lim_{t \to \infty} \left( Y(0) \frac{g - \tau}{r - n} e^{(n-r)t} + Y(0) \left( b(0) - \frac{g - \tau}{r - n} \right) \right) = 0$$

holds. Since in the typical case<sup>4</sup> (Romer D. 1996)r > n, the above condition is satisfied only when the second component equals 0, i.e. when  $g - \tau = b(0)(r - n)$ . As the equation shows, it is true only when  $\dot{b} = 0$ . Thus, the ratio of the primary expenditure to GDP may be maintained at the fixed level only when the relative level of the public debt also remains fixed, so the socially optimum level of deficit in this case equals  $d_s^* = (n + \pi)b$ . The result is the same as the one obtained in the case of the utility function based approach.

Under the fixed relative state income, it implies that the ratio of service costs to GDP has to be fixed, therefore the relative deficit level fulfilling the assumption has to be equal to the product of the ratio of public debt to GDP and the nominal economic growth rate. This result is identical with the one achieved under the assumption based on the utility function.

To summarize, in both cases it was proved that the socially optimal government policy is based on maintaining the fixed relative deficit d at the level equal to the product of public debt level in relation to GDP and the nominal GDP growth rate. As a result of such a policy the ratio of the public debt value to GDP will remain at the fixed level.

<sup>&</sup>lt;sup>4</sup> As Romer indicates assuming that the long run interest rate is higher than the long run economic growth rate is equal to assumption that the analysed economy is dynamically efficient in the long run.

#### 3. Reasons and consequences of the excessive deficit

This section concentrates on the analysis of long run consequences of the case in which the real deficit deviates from what is defined as socially optimal. With given tax income, it is synonymous with a situation in which the current expenditure level is higher than the one providing the maximization of the long run social utility. Milesi-Ferretti (1997) indicates that such a phenomenon is relatively common in democratic countries. Although the analysis of its origins goes beyond this study, it is worth mentioning at least several possible causes of the deficit bias.

- Models of fiscal illusion suggest that the government, instead of maximizing the social welfare, can be driven by maximizing the possibility of winning the election. In the case in which the society does not fully realize the long run consequences of the chosen policy, it may lead to votes "buying" which causes the deficit to increase above the optimum level.
- According to models that treat debt as a strategic variable, the existing public debt is a variable which has an impact on behaviour of the future governments, thus the existing government may to a certain extent use the deficit to shape the desired resources allocation.
- A conflict between social groups of interest (e.g. political parties) often leads to lags in introducing reforms which aim at reducing the growth of expenditure.
- Models that put the emphasis on the problem of wealth redistribution between generations may suggest that the generation which is currently in power will favour the current expenditures as opposed to the future ones.

Let us assume that the objectives of the government's activity may diverge from the objectives of the society as a whole. On the one hand, the government strives to satisfy the electors who aim at maximizing the utility in the long run. On the other hand, it may believe that there is a possibility of gaining popularity and chances to be re-elected by increasing the current expenditure. Let the government's objective be minimising the loss function

(24)  $L = \frac{1}{2}(d - d_s^*)^2 - \varphi d$ ,

where  $\varphi > 0$  represents a relative importance the government attaches to the efforts of gaining popularity by increasing the current expenditure and, consequently, the deficit. If this importance is close to 0, then the activities of fiscal authorities are concentrated on achieving the socially optimal solution. The deficit level minimizing the given loss function is

# (25) $d_B^* = d_S^* + \varphi$ .

Thus, the greater impact on maximizing the current government expenditure causes the excessive deficit. As a consequence of maintaining such a deficit the debt trap (Ponzi-type outcome) occurs. It should be noticed that  $d_s^*$  defines the maximal level at which the relative public debt level will not increase in a certain period. Increasing the deficit above this level by a fixed value  $\phi$  causes the debt to increase unlimitedly. Even if it is possible to maintain the excessive deficit for a certain period, such policy will end up with the country's insolvency, with all its subsequent negative consequences.

The excessive deficit as it is presented above does not satisfy the intertemporal budget constraint. However, we can imagine an alternative situation where despite an excessive deficit, the relative debt level does not increase in an unlimited way. It has to be assumed that there exists a mechanism constraining the maximal deficit level. The source of this constraint may the capital markets, the participants of which assess the policy and, being driven by rational expectations, refuse to finance under the debt trap conditions. In the presented analysis it is assumed that in the long run such mechanism imposes a constraint on the country's financing and thus the long run relative deficit level cannot be endlessly high. Let us assume that this constraint is known and fixed in categories of the deficit-to-GDP ratio and equals  $\hat{d}_F$ . In this situation, the real deficit level will be  $d = \min(d_B^*, \hat{d}_F)$ . To simplify it, let us assume that financial markets would be able to provide the deficit financing, given that the deficit remains on the socially optimal level, which can be expressed as  $d_S^* < \hat{d}_F$ .

Consequently, the relative budget deficit will remain at the fixed level  $\hat{d}_F$ , which causes the fixed debt growth. It may be proved that, unlike in the previously considered example, the debt growth is not explosive this time. According to the equation, the growth of relative debt level equals the difference between the relative deficit and the product of relative debt and nominal GDP growth rate. As the debt rises, the difference decreases, which causes slowdown of the growth of the relative debt level. In a very long run (with t $\rightarrow\infty$ ) this growth converges to 0:  $\lim_{t\to\infty}(\hat{d}_F - (n + \pi)b) = 0$ . The public debt

<sup>&</sup>lt;sup>5</sup> This assumption does not affect the generality of the solution. If at a period t=0 this condition is not satisfied, then such a situation leads to growth of the relative public debt level. This, in turn, causes the growth of the socially optimal debt level  $b_s^*$  and consequently growth of  $b_B^*$  as well. Hence, there exists such a period t' for which for each t > t' the discussed condition will be satisfied.

in ratio to GDP will asymptotically converge towards the following equilibrium level:

(26)  $\hat{b}_{L} = \hat{d}_{F} / (n + \pi)$ .

To illustrate the discussed process, a simple simulation was carried out. Macroeconomic parameters of the model economy were assumed to be the same as in the previous numerical example. Moreover, it was assumed that the maximal deficit level  $\hat{d}_F$  possible to finance in the long run is 3% of GDP. Paths of the most important fiscal variables (the public debt, deficit and public expenditure g) are presented in Figure 1.

# Fig. 1 The results of simulation of public debt (b), deficit(d) and primary expenditure (g) as a share of GDP\*



\* LS (RS) denotes the variable presented on the left (right) scale. Source: Author's calculations.

Let us assume that the relative deficit during the whole simulation period is constant and equals  $\hat{d}_F = 3\%$ . This level exceeds the socially optimal deficit value which, as it was proved in the previous example, initially equals 1,65% of GDP. The excessive deficit causes the relative public debt level to rise asymptomatically towards the long run level  $\hat{b}_L = 75\%$ . As Fig. 1 presents, during the analysed period of 50 years the variable reaches the approximate level of about 70% of GDP. The public debt growth causes the relative expenditure level to vary and deviate from the optimal path. Initially the expenditure share in GDP was  $g = \tau + \hat{d}_F - (r + \pi)b = 40,6\%$ , so it was higher than the optimum 39%, however it decreases with time when the interest on the increasing debt rises. In a very long run the expenditure level asymptotically converges towards  $g_L = 38,5\%$ , a level that is lower than the one which would be the result of the socially optimal fiscal policy.

To summarize, it appears that the discussed case concerning the excessive deficit gives a limited long run solution and does not result in debt trap. However, it does not mean that such practices are optimal since the assumed deficit differs from the optimum solution by the difference of  $\delta = \hat{d}_F - d_S^*$ . Pursuing the policy of constant excessive deficit means that a part of future consumption is replaced by the current one. The higher current expenses cause an additional debt growth which in the future will cause a growth of interest payments and decrease of consumption. Such a policy causes the social utility level to decrease.

#### 4. Comparison of fiscal policy rules

As it was mentioned at the beginning, institutional factors can play an important role in shaping fiscal policy, as well as reducing the excessive deficit. The cited authors mention the fiscal policy rules among the most important institutions. Applying these rules aims at obliging the government to conduct a policy which would be closest to the socially optimal one.

The subject of this section is an attempt to compare some most common types of fiscal rules and their optimality in the long run. There can be at least several criteria of the rules assessment – the detailed list can be found among others in papers by Inman (1996), Kopits and Symansky (1998), Buti and Giudice (2002) or Buiter (2003). Here we will concentrate on only two criteria which are chosen because of their crucial long run importance. These are:

- compliance of a given rule with the socially optimal solution;
- simplicity and convenience of application, including also immunity to the lack of complete and detailed information.

The importance of the first criterion was discussed earlier in this study – the primary objective of the application of fiscal rules is to change the current fiscal policy to bring it at least a little closer to the socially optimal one. The second criterion has only a technical character, however in practice it may turn out to be crucial. In the light of this condition one should favour a rule the application of which requires the knowledge of only one or two observable parameters, instead of a rule which demands detailed long run forecasts of a number of economic variables.

First of the rules discussed is the **deficit rule**, according to which the deficit in a given period should not exceed a certain level. As its most common version says, the deficit should in no period exceed a certain fixed level, expressed as a percent of  $GDP^6$ .

 $(27) \qquad d \le \hat{d}_R \,.$ 

According to earlier considerations (see equation ), the deficit level which meets the requirements of the socially optimal policy is:

(28) 
$$d_{R} = (n + \pi)b$$
.

In the light of the discussed model, accepting any other deficit level would be sub-optimal in the long run. The constraint set at the level higher than the product of nominal growth rate and relative debt level would favour, from the point of view of the social utility, too much the current consumption . Introducing a constraint which is too low (e.g. the balanced budget rule) would, in turn, lead to an excessive reduction of the current consumption in favour of its future level.

However, applying such a rule involves some difficulties in practice. Though the public debt value in relation to GDP is known, it is often hard to define the long run GDP growth rate. Defining a rate which is too high will lead in an extreme case ( and the lack of corrective actions) to an explosive path of deficit and the debt trap. It is connected with the fact that in such a case the deficit would be defined by the formula  $d = (n + \pi + \nu)b = d_s^* + \nu b$ , where  $\nu$  denotes error in the assessment of the long run rate growth. With  $\nu > 0$  the deficit would equal  $d_s^*$  value which provides the stabilization of public debt, increased by a certain value proportional to the relative debt level. The effect of policy conducted according to such a rule would be an accelerating debt growth.

<sup>&</sup>lt;sup>6</sup> According to the optimal rule the deficit should , in fact, be exactly equal to the product of nominal growth rate and the relative debt. However, it may be assumed that the government has no reasons to pursue a more restrictive policy than it is acceptable (the lower deficit policy). Thus, a more practical deficit ceiling is sufficient.

A way to avoid this problem would be defining in the period t = 0 the deficit ceiling  $d_s^*$  which could not be corrected when the relative debt level changed. In a case when  $\overline{\hat{d}}_R > d_s^*$  an identical situation would occur as in the case of setting the ceiling at  $\hat{d}_F$ . The relative debt level would start to rise, approaching asymptotically the new equilibrium level compliant with the higher deficit. In a case where  $\overline{\hat{d}}_R < d_s^*$  the relative debt would asymptotically decrease towards the new, lower long run level. Although such a solution allows to avoid the debt trap problem, still its effectiveness concerning achieving the main objective, i.e. the socially optimal policy, depends on the knowledge of the long run economic growth rate. Applying the deficit rule leads to the socially optimal solution only when in the period t = 0 the rate is estimated correctly.

The **modified deficit rule** is devoid of these drawbacks. According to this rule in each period the deficit should be set at the level which causes the relative debt level to remain constant:

(29)  $d \le \hat{d}'_R$ , where  $\hat{d}'_R : \dot{b} = 0$ 

Applying this rule does not require knowledge of the long run parameters. It only requires a constant monitoring of the debt level and current corrections of expenditure so that the public debt-to- GDP ratio remains constant. This solution, with the assumption that the monitoring system is efficient enough, allows to achieve the socially optimal fiscal policy, without any risk in the long run.

This rule is, in fact, similar to another widely used class of fiscal rules – **public debt rules**. Their typical version assumes that the debt level should not exceed a certain defined value. Most often the marginal debt value is defined in relative categories, i.e. as a percentage of GDP:

$$(30)$$
  $b \leq b_R$ .

According to the earlier considerations, this rule should provide the socially optimal policy only when the maximal debt level is defined at the initial level, i.e.  $\hat{b}_R = b(0)$ . A rule defined in such a way is identical with the presented modified deficit rule.

In practice, however, the maximal public debt level is often defined at the level which is higher than the current one, i.e.  $\hat{b}_R > b(0)$ . An example may be the solution applied in Poland, according to which the public debt level should

not exceed 60% of GDP, while the current debt level is about 50% of GDP<sup>7</sup>. Setting the debt ceiling at a level other than the current one does not comply with the optimality requirement. It is easiest to trace it on the following numerical example.

All economy parameters are assumed at the same level as in the simulation presented earlier. It was assumed that there exist a fiscal rule, according to which the public debt cannot exceed the level  $\hat{b}_R = 60\%$ . Time paths of the most important fiscal variables in the simulation are presented in Figure 2.



# Fig. 2 Results of simulation of public debt (b), deficit (d) and public expenditure (g) in relation to GDP in the case of applying public debt fiscal rule

Source: Author's calculations.

As long as the public debt does not exceed the 60% of GDP, trajectories of all the above fiscal variables are identical with the ones presented in the earlier simulation. As soon as the relative level of the public debt reaches 60%, there is a necessity for sharp reduction of the deficit from the initial 3% of GDP to the new, lower level  $d_L = (n + \pi)\hat{b}_R = 2,4\%$ , which allows to stabilize of the public debt on  $\hat{b}_R$  level. It means the necessity to cut the expenditure to the level of

<sup>&</sup>lt;sup>7</sup> At the end of year 2002, source: Ministry of Finance

 $g_L = \tau + d_L - (r + \pi)\hat{b}_R = 38,8\%$ . This level is higher than in the fully unrestricted case (expenditure on the level of 38,5% of GDP), while still lower than the one which is the result of the socially optimal policy (39,2%).

We should notice yet another aspect connected with applying this rule, though it goes beyond the framework of the presented simple model. This rule brings the pursued policy closer to the socially optimal solution, however, it occurs at cost of necessity of sharp cut of the expenditure , thus disturbing its smooth path. As Belka (2003) indicates, this correction is very socially costly and may lead to abandonment of the rule , which undermines its sense.

The **expenditure rules**<sup>8</sup> are the last analyzed category. Such typical rule assumes that the country's expenditure will not rise at the rate higher than a certain ceiling value. It most often concerns the overall expenditure of the public sector  $G_T$ , given by  $G_T = G + (r + \pi)B$ . As previously, we assume that the government is willing to use the possibility to increase expenditure, provided that this action does not violate the rule. According to this assumption, the expenditure will actually rise at the ceiling rate. If the rate was different from the nominal GDP growth rate, the ratio of the government expenditure to GDP in the long run would decrease to 0 or rise to 100% level. Maintaining the approximate share of the government sector expenditure in GDP requires the rate of the nominal growth rate. Therefore, the rule of expenditure concerning the long run is expressed as:

 $(31) \qquad \dot{G}_{T} / G_{T} \leq n + \pi$ 

Whether or not the so defined rule will guarantee the socially optimal policy will depend on the deficit level in the period 0. Setting the fixed rate of expenditure growth means that in the long run the ratio of  $G_T$  to GDP will remain constant. The fixed tax rate means the fixed relative deficit level. Thus, the long run effect of the rule allows to maintain the fixed relative deficit at the same level as in the period t = 0. Therefore, if initially the deficit is at the socially optimal level, then the result of applying the expenditure rule will be the socially optimal solution in the long run. If this condition is fulfilled,

then the result of the fixed relative deficit will be convergence of the relative public debt to a certain new long run level, calculated

<sup>&</sup>lt;sup>8</sup> A broader analysis concerning the practical aspects of applying the expenditure rules is presented by Mills P., Quinet A. 2001, *The Case for Spending Rules*, [in:] Banca d'Italia, 2001, *Fiscal Rules*, proceedings from the conference held in Perugia, February 2001.

analogically as in the equation 3. This solution is not socially optimal, however it allows to avoid the debt trap situation - in a very long run it leads to the stabilization of the relative public debt level.

In the case of the expenditure rule, it is crucial to choose the right nominal rate of expenditure growth. Even minor deviations of the rate from the long run product growth rate cause (if no corrections are made) a rapid growth or decrease of the relative expenditure level to the level of 100% or 0%, respectively.

To avoid this problem, the **modified expenditure rule** may be applied, according to which the rate of expenditure growth should equal a certain moving average of the past expenditure growth rates. The advantage of this rule may be not evident enough in the presented model, since the long run rate of economic growth is fixed and known here. However, in practice it is hard to predict, which is especially important in such countries as Poland, where the available time series are short. Applying the modified expenditure rule eliminates the problem to a great extent as it only requires the knowledge of historical data. It does not mean, however, that the fiscal policy compliant with this rule is automatically optimal, since fulfilling the optimality condition also requires the appropriate initial deficit level. Thus, the expenditure growth rules should be applied together with some other rule which will provide the optimum deficit level in the period 0. Only then do these rules guarantee the policy that is socially optimal in the long run.

#### Summary and conclusions

The performed analysis may be summarized in the following way:

- The subject of the analysis is the long run model of the public finance sector, with the intertemporal choice and infinite time horizon. In the basic model we try to find a policy which provides the optimum distribution of expenditure in time. Two alternative optimality criteria are considered: the one based on the utility function and the other, based on assuming constant share of public expenditure in GDP. Moreover, the optimal solution also has to satisfy the long run budgetary constraint, according to which the sum of the future discounted income has to allow for financing the sum of the future discounted expenditure and the initial public debt.
- It is shown that the solution is the policy that maintains such a level of expenditure and deficit which provides the stabilization of the public debt-to-GDP ratio.

- It is assumed that there are reasons for which the real fiscal policy may deviate from the socially optimal policy. The reason that the deficit is higher than the social optimum may be "fiscal illusion", a conflict between the interest groups or between generations. Such a policy causes a loss in the social utility, and in the extreme case can end up with an uncontrolled growth of the public debt and insolvency of the public sector.
- A considered method to reduce the unfavorable activities of government involves applying the fiscal policy rules. Three groups of rules are analyzed, that refer to three crucial fiscal variables: deficit, public debt and expenditure.
- The first analyzed group are the deficit rules, that impose a constraint on the deficit-to-GDP ratio. Among the three considered versions, the modified deficit rule seems to be the most promising. It involves the monitoring of the public debt level and such a choice of deficit that the debt-to-GDP ratio remains constant. Applying this rule does not require knowledge of the long run economic growth rate, and also guarantees that the chosen fiscal policy satisfies both the social optimality condition and the intertemporal budget constraint.
- Another analyzed group of rules are the debt ceilings, setting a constraint on the debt-to-GDP ratio. Their effectiveness in fulfilling the objectives depends on the level at which the ceiling is set. The debt rules guarantee optimal policy only when the defined constraint equals the initial ratio of the debt to GDP in this case the debt rule brings the same results as the modified deficit rule. If the debt constraint is defined at a level higher than the present one, then the result will be a excessively lax fiscal policy. As soon as the debt growth makes constraint binding, the rule forces a correction being a sharp reduction of the expenditure, which is almost always unfavorable from the point of view of social utility.
- The last solution considered are the expenditure growth rules, the effects of which involves introducing the permissible rate of the overall expenditure growth. If the path is to make sense in the long run, this rate has to be the same as the long run GDP growth rate, which makes this method very sensitive to any possible estimation errors. The *modified* expenditure rule is devoid of this drawback according to it the expenditure growth rate should equal a moving average of the past income growth rates. This approach does not require the knowledge of the long run economic growth rate but only observing the real income growth rate and making appropriate adaptations of the expenditure growth rate. However, if applying this rule is to lead to the socially optimal policy, it is necessary that another condition is fulfilled –

the initial deficit level has to be socially optimal, i.e. it cannot cause the increase of the public debt-to-GDP ratio.

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